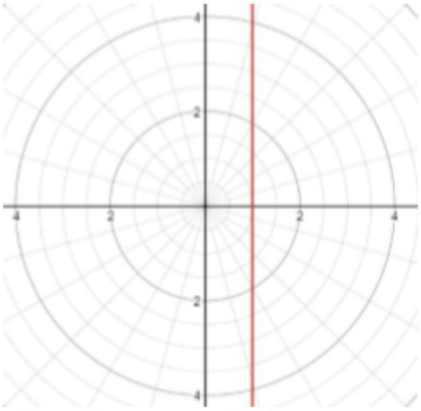
Polar Equations of Lines

Vertical Line

$$r\cos(\theta) = a$$



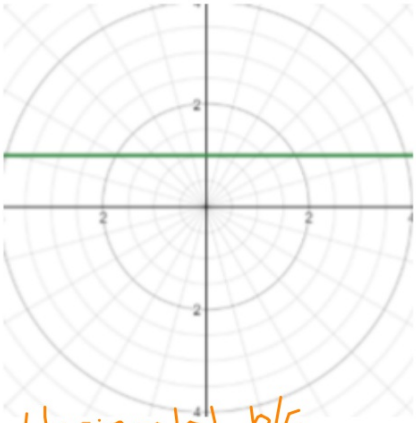
Vertical b/c

$r\cos\theta = x$

$x = a$ is vertical

Horizontal Line

$$r\sin(\theta) = b$$



Horizontal b/c

$r\sin\theta = y$

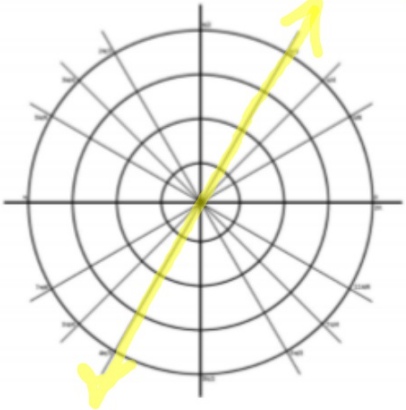
$y = b$

Sloped Line

$$\theta = \beta$$

ex.)

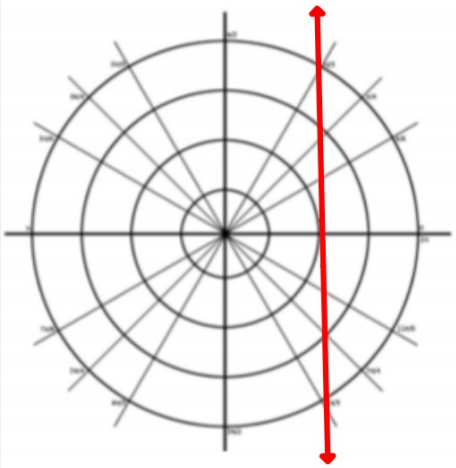
$\theta = \frac{\pi}{3}$



Polar Equations of Lines

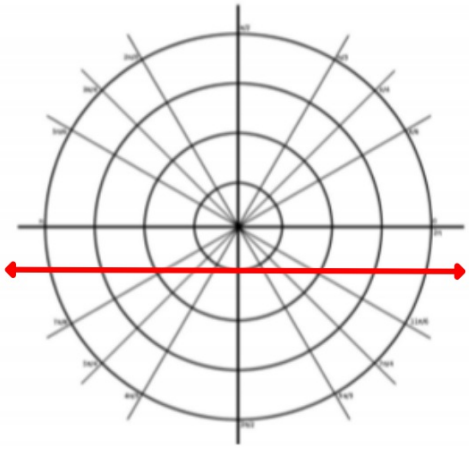
Vertical Line

$$r\cos(\theta) = 2$$



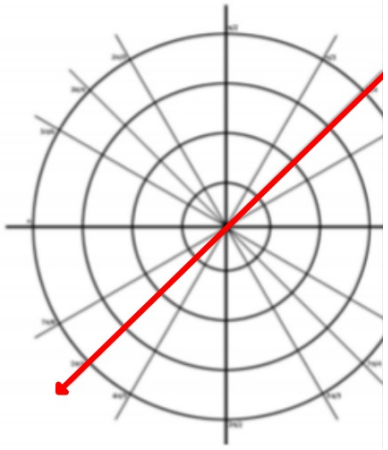
Horizontal Line

$$r\sin(\theta) = -1$$



Sloped Line

$$\theta = \frac{\pi}{4}$$



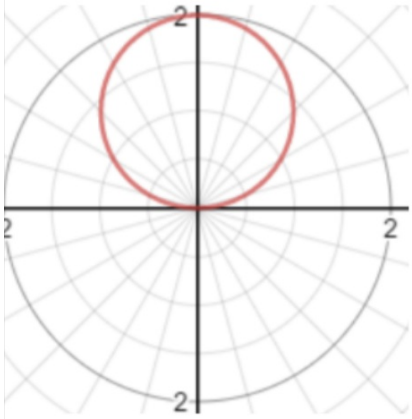
Polar Equations of Circles

-2b : underneath on y-axis

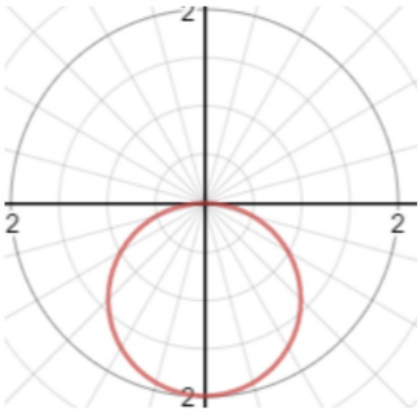
Vertical Circle

$r = 2b\sin(\theta)$ on y-axis, 2b is pos - up

2b = diameter



$r = -2b\sin(\theta)$



Polar Equations of Circles

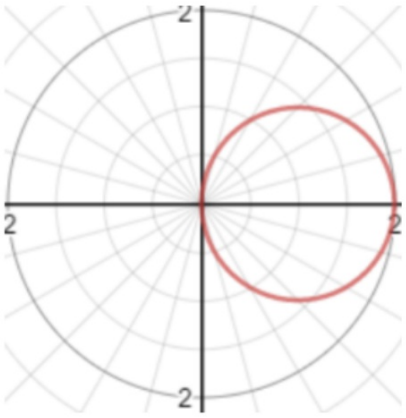
Horizontal Circle

$r = 2a\cos(\theta)$ → on x-axis

positive

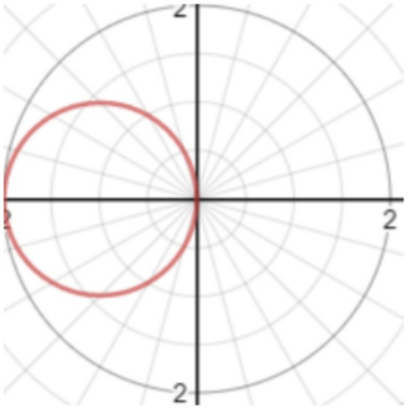
2a → right

2a : diameter



$r = -2a\cos(\theta)$

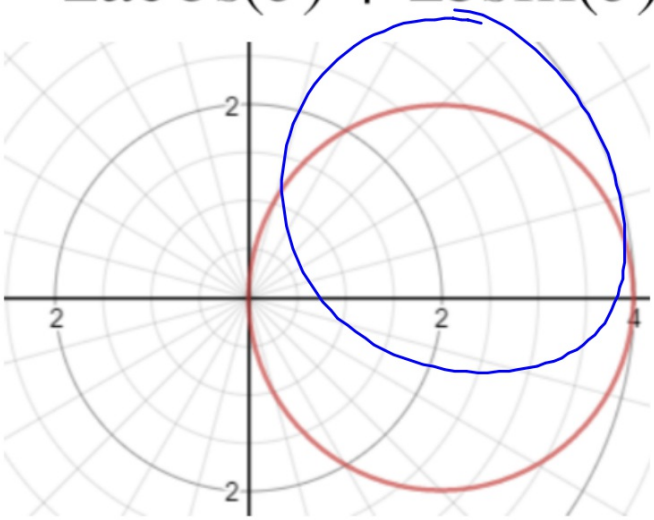
-2a: on x-axis on left



Circle Away from the Pole

center is not on an axis

$$r = 2a\cos(\theta) + 2b\sin(\theta)$$

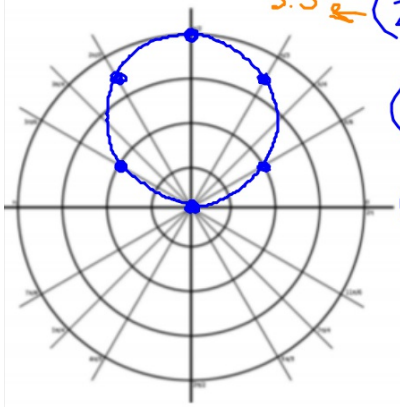


Polar Equations of Circles

(r, θ) plug in θ, get r

Vertical Circle

$r = 4\sin(\theta)$



$(0,0)$

$(2, \frac{\pi}{6})$

3.5 $(2\sqrt{3}, \frac{\pi}{3})$

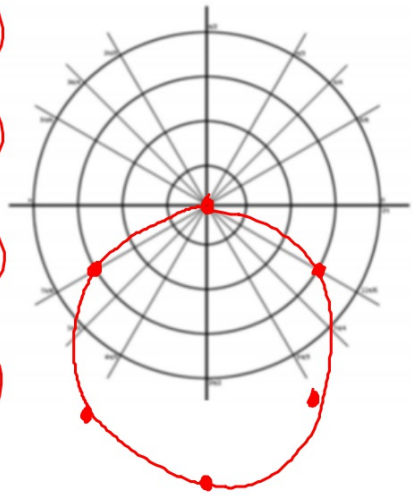
$(4, \frac{\pi}{2})$

$(2\sqrt{3}, \frac{2\pi}{3})$

$(2, \frac{5\pi}{6})$

$(0, \pi)$

$r = -6\sin(\theta)$



$(0,0)$

$(-3, \frac{\pi}{6})$

5.2 $(-3\sqrt{3}, \frac{\pi}{3})$

$(-6, \frac{\pi}{2})$

$(-3\sqrt{3}, \frac{2\pi}{3})$

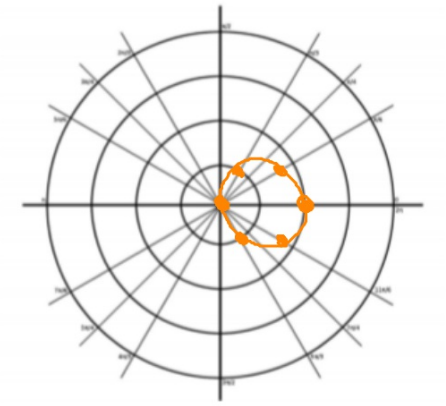
$(-3, \frac{5\pi}{6})$

$(0, \pi)$

Polar Equations of Circles

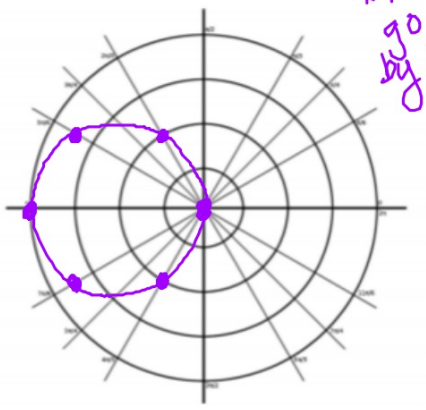
Horizontal Circle

$r = 2\cos(\theta)$



(r, θ)	
$(2, 0)$	$(-8, 0)$
$(\sqrt{3}, \frac{\pi}{6})$	$(-4\sqrt{3}, \frac{\pi}{6})$
$(1, \frac{\pi}{3})$	$(-4, \frac{\pi}{3})$
$(0, \frac{\pi}{2})$	$(0, \frac{\pi}{2})$
$(-1, \frac{2\pi}{3})$	$(4, \frac{2\pi}{3})$
$(-\sqrt{3}, \frac{5\pi}{6})$	$(4\sqrt{3}, \frac{5\pi}{6})$
$(-2, \pi)$	$(8, \pi)$

$r = -8\cos(\theta)$

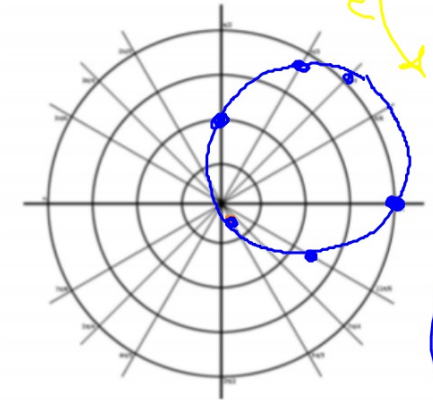


*radii go by 2's

Circle Away from the Pole

rcle with Sine and Cosine

$= 4\cos(\theta) + 2\sin(\theta)$



3. 1. 2.

same

$(4, 0) \rightarrow 4 + 0$

$\left(2, \frac{\pi}{2}\right) \rightarrow 0 + 2$

$(-4, \pi) \rightarrow -4 + 0$

2 points

$\left(3\sqrt{2}, \frac{\pi}{4}\right) \rightarrow 2\sqrt{2} + \sqrt{2}$

3.7

$\left(2\sqrt{3}, \frac{\pi}{3}\right) \rightarrow 2 + \sqrt{3}$

2.5

$\left(2\sqrt{3} - 1, \frac{11\pi}{6}\right) \rightarrow 2\sqrt{3} - 1$

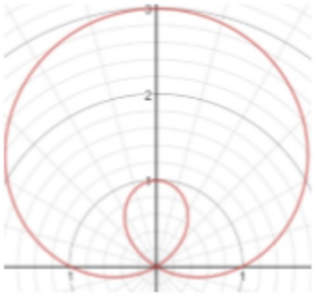
.3

$\left(2 - \sqrt{3}, \frac{5\pi}{3}\right) \rightarrow 2 - \sqrt{3}$

Limacons - Inner Loop

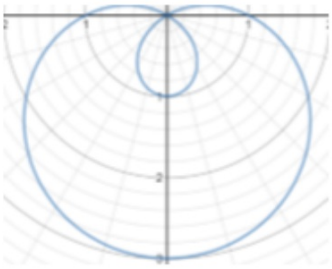
on y-axis above

$r = a + b\sin(\theta)$



on y-axis below

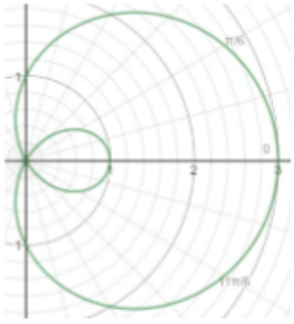
$r = a - b\sin(\theta)$



$\frac{a}{b} < 1$

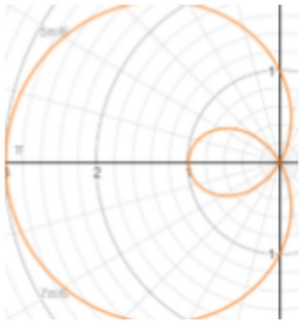
on x-axis right

$r = a + b\cos(\theta)$



on x-axis left

$r = a - b\cos(\theta)$



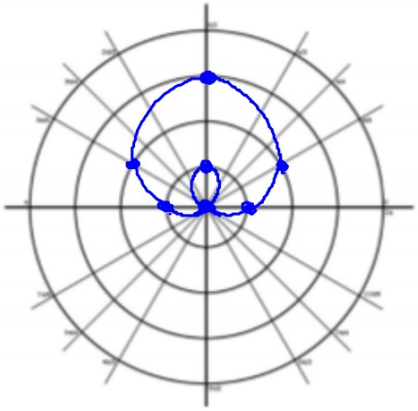
$\frac{a}{b} < 1$

Quick Tips for Graphing Limacons

- $||a|-|b||$ distance for inner loop
- $|a|+|b|$ distance for outer loop
- a is x or y intercepts depending on orientation
- Choose four cardinal points and all multiples of $\frac{\pi}{6}$ for graphing sine equations $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
- Choose four cardinal points and all multiples of $\frac{\pi}{3}$ for graphing cos functions

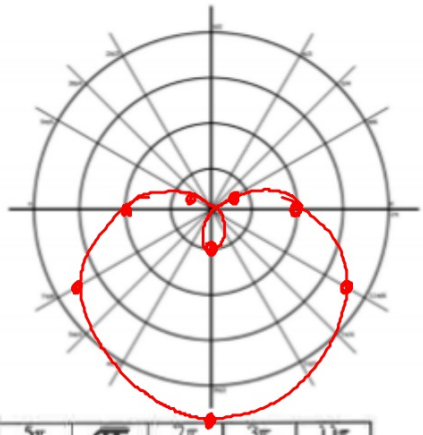
Limacons - Inner Loop

$r = 1 + 2\sin(\theta)$



$\frac{a}{b} < 1$

$r = 2 - 3\sin(\theta)$



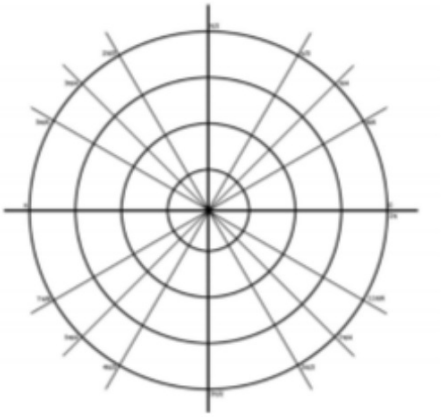
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
r	1	2	3	2	1	0	-1	0

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
r	2	$\frac{1}{2}$	-1	$\frac{1}{2}$	2	3.5	5	3.5

(r, θ)

$(1, 0)$

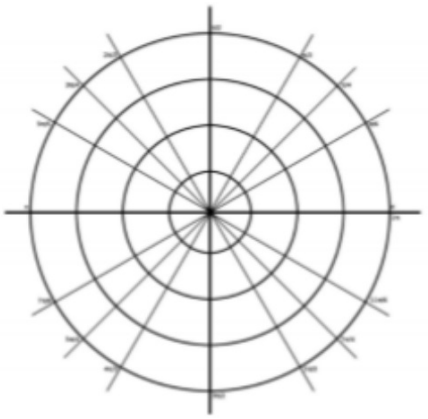
$r = 1 + 3\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$

$\frac{a}{b} < 1$

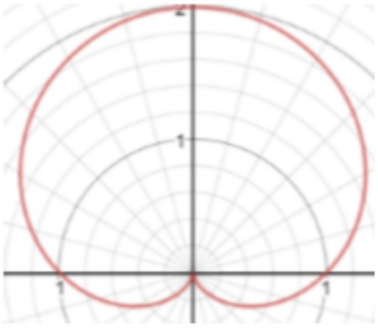
$r = 3 - 4\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$

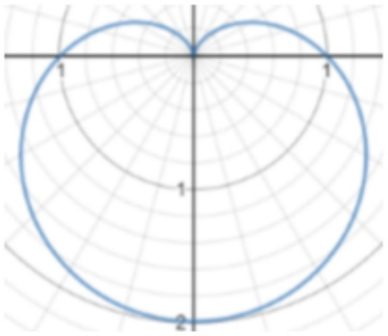
Cardioids

$r = a + b\sin(\theta)$

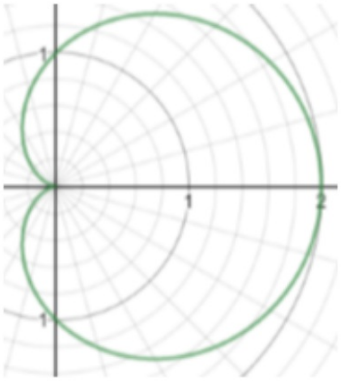


$\frac{a}{b} = 1$

$r = a - b\sin(\theta)$

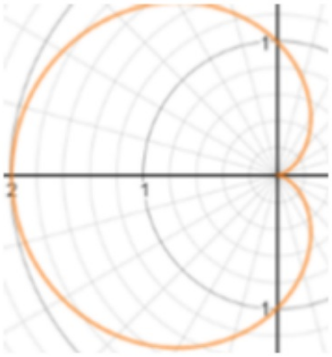


$r = a + b\cos(\theta)$



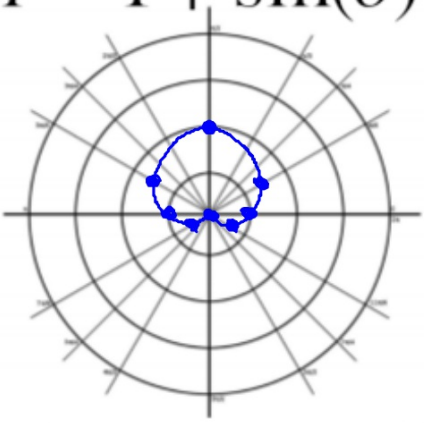
$\frac{a}{b} = 1$

$r = a - b\cos(\theta)$



Cardioids

$r = 1 + \sin(\theta)$



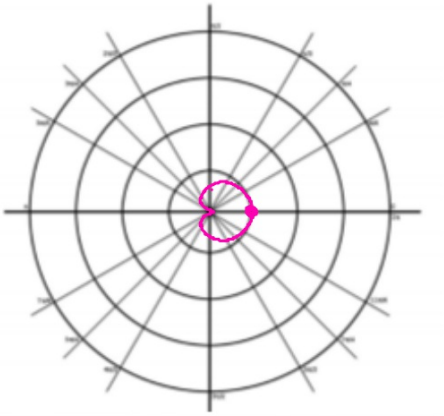
$\frac{a}{b} = 1$

$r = 2 - 2\sin(\theta)$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
r	1	1.5	2	1.5	1	.5	0	.5

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
r	2	1	0	1	2	3	4	3

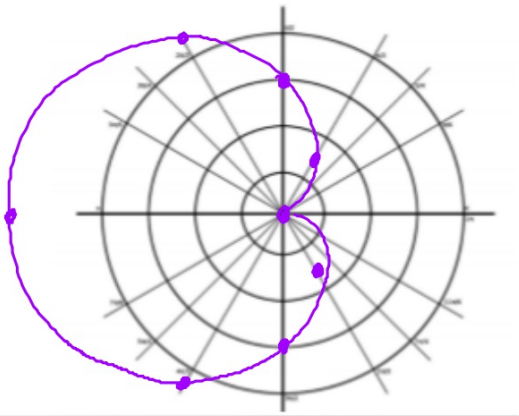
$$r = \frac{1}{2} + \frac{1}{2}\cos(\theta)$$



θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
r	1	.75	.5	.25	0	.25	.5	.75

$$\frac{a}{b} = 1$$

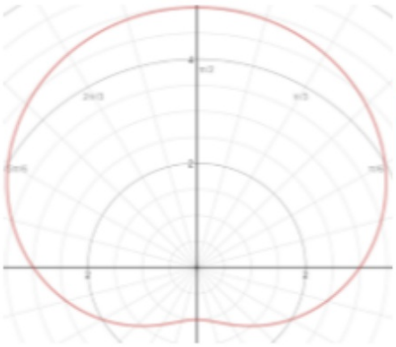
$$r = 3 - 3\cos(\theta)$$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
0	1.5	3	4.5	6	4.5	3	1.5

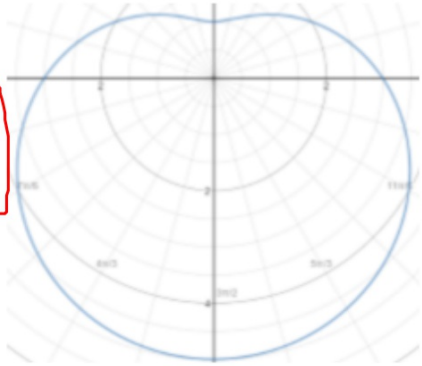
Limacons - Dimpled

$r = a + b\sin(\theta)$

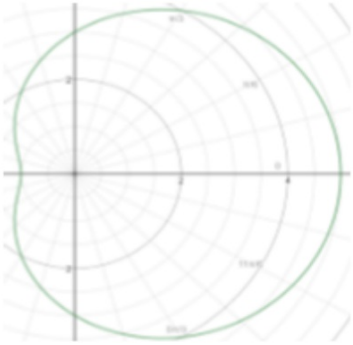


$1 < \frac{a}{b} < 2$

$r = a - b\sin(\theta)$

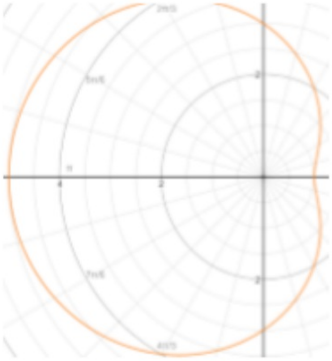


$r = a + b\cos(\theta)$



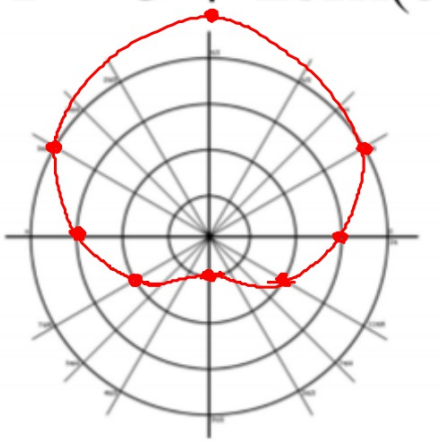
$1 < \frac{a}{b} < 2$

$r = a - b\cos(\theta)$



Limacons - Dimpled

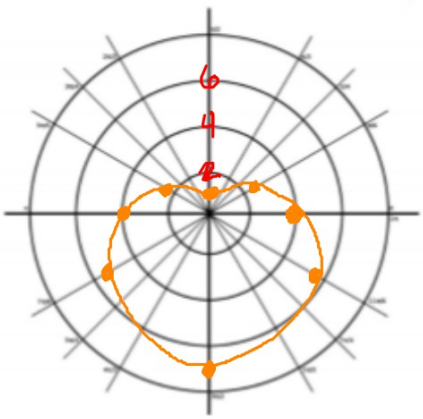
$r = 3 + 2\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
3	4	5	4	3	2	1	2

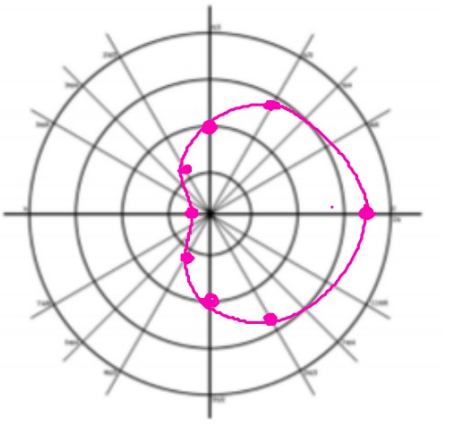
$1 < \frac{a}{b} < 2$

$r = 4 - 3\sin(\theta)$ ★ Go by .5



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
4	2.5	1	2.5	4	5.5	7	5.5

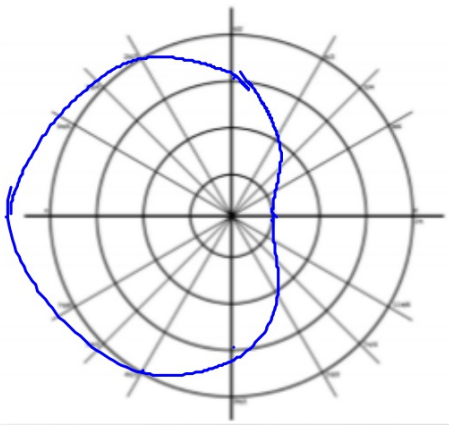
$r = 4 + 3\cos(\theta)$ *Go by 2's



θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
r	7	5.5	4	2.5	1	2.5	4	5.5

$1 < \frac{a}{b} < 2$

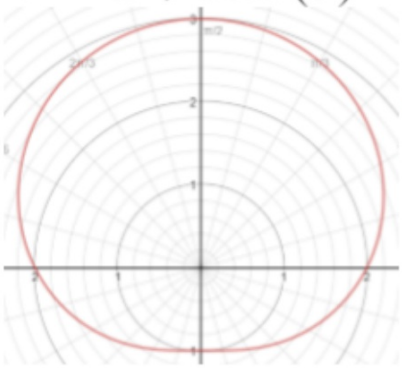
$r = 3 - 2\cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
1	2	3	4	5	4	3	2

Limacons - Convex

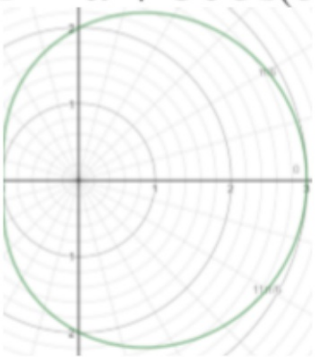
$r = a + b\sin(\theta)$



$r = a - b\sin(\theta)$

$\frac{a}{b} \geq 2$

$r = a + b\cos(\theta)$

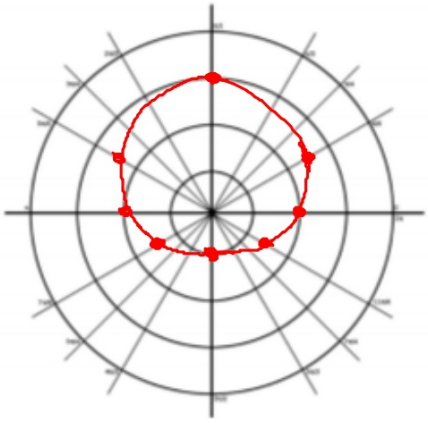


$r = a - b\cos(\theta)$

$\frac{a}{b} \geq 2$

Limacons - Convex

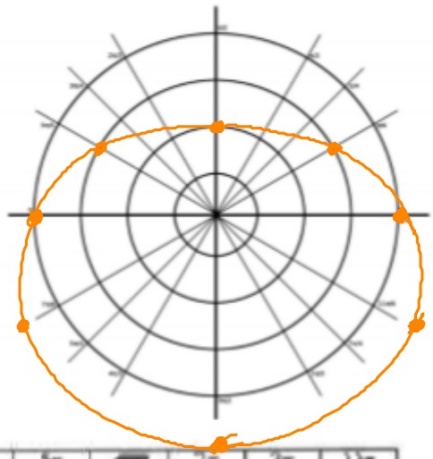
$r = 2 + \sin(\theta)$



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
r	2	2.5	3	2.5	2	1.5	1	1.5

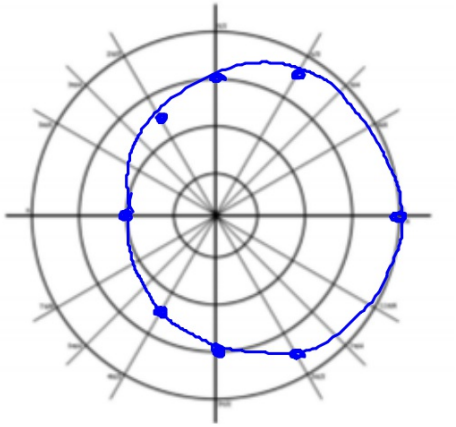
$\frac{a}{b} \geq 2$

$r = 4 - 2\sin(\theta)$



0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
4	3	2	3	4	5	6	5

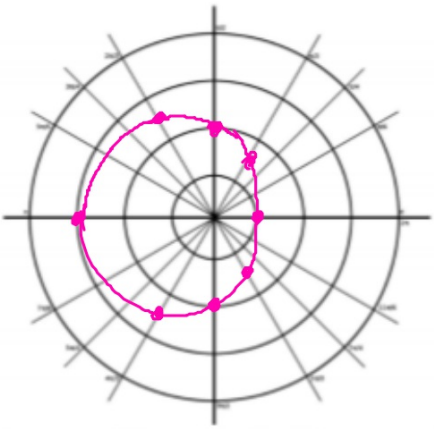
$r = 3 + \cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
4	3.5	3	2.5	2	2.5	3	3.5

$\frac{a}{b} \geq 2$

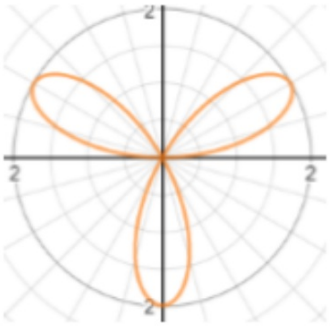
$r = 2 - \cos(\theta)$



0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
1	1.5	2	2.5	3	2.5	2	1.5

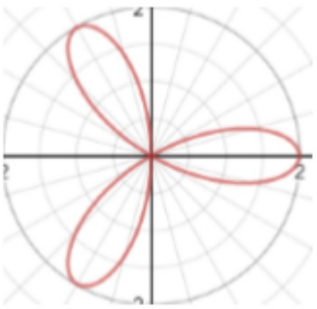
Rose Petals

$r = a\sin(n\theta)$

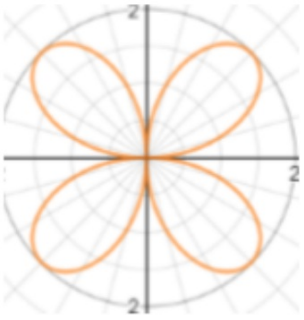


n is odd

$r = a\cos(n\theta)$

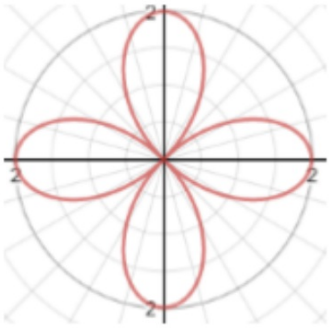


$r = a\sin(n\theta)$



n is even

$r = a\cos(n\theta)$



Quick Tips for Graphing Rose Petal Curves

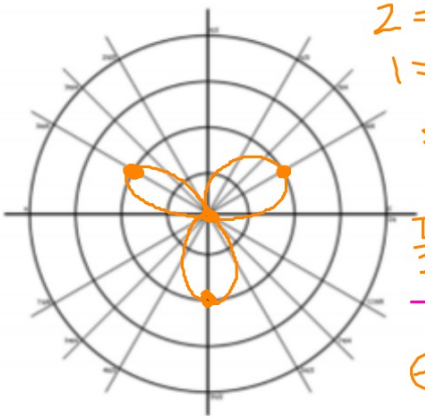
- If n is odd same number of petals
- If n is even double the number of petals
- a is the length of a petal
- Choose four cardinal points and determine the period to aid in choosing points

- If n is odd → add 2π, then divide by n
- If n is even → add π, then divide by n

Rose Petals

length 2, 3 petals

$$r = 2\sin(3\theta)$$



$2 = 2\sin(3\theta)$

$1 = \sin(3\theta)$

$\sin^{-1}(1) = 3\theta$

$\frac{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}}{3} = \frac{3\theta}{3}$

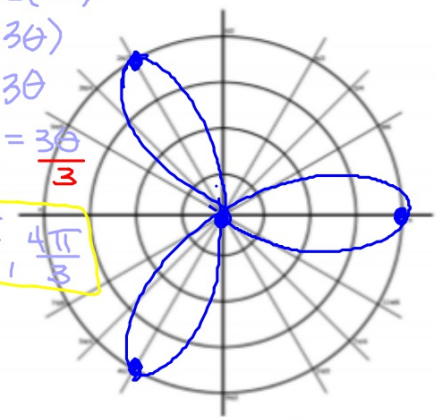
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

θ	0	$\pi/2$	π	$\frac{3\pi}{2}$		$\pi/6$	$5\pi/6$	
r	0	-2	0	2		2	2	

only 2 pts

length 4, 3 petals

$$r = 4\cos(3\theta)$$



$4 = 4\cos(3\theta)$

$1 = \cos(3\theta)$

$\cos^{-1}(1) = 3\theta$

$\frac{0, 2\pi, 4\pi}{3} = \frac{3\theta}{3}$

$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

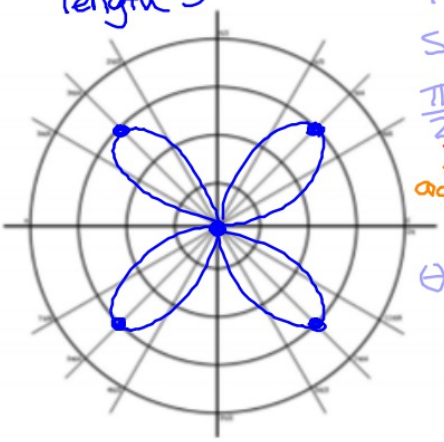
n is odd

θ	0	$\pi/2$	π	$3\pi/2$		$2\pi/3$	$4\pi/3$	
r	4	0	-4	0		4	4	

only 2 pts

$r = 3\sin(2\theta)$ → 4 petals

length 3



$3 = 3\sin(2\theta)$

$1 = \sin(2\theta)$

$\sin^{-1}(1) = 2\theta$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} = 2\theta$

add π → $\div 2$

n is even

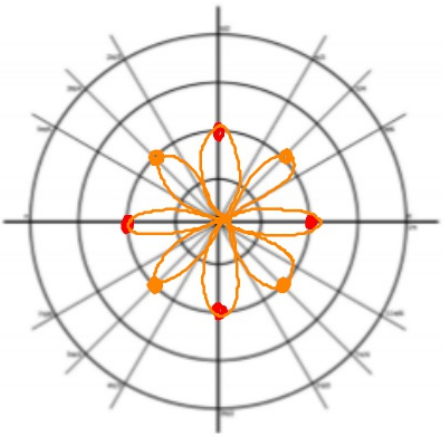
$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

θ	0	$\pi/2$	π	$3\pi/2$	$\pi/4$	$3\pi/4$	$5\pi/4$	$7\pi/4$
r	0	0	0	0	3	-3	3	-3

One point

$r = 2\cos(4\theta)$ → 8 petals

length 2



θ	0	$\pi/2$	π	$3\pi/2$	$\pi/4$	$3\pi/4$	$5\pi/4$	$7\pi/4$
r	2	2	2	2	-2	-2	-2	-2

$2 = 2\cos(4\theta)$

$1 = \cos(4\theta)$

$\cos^{-1}(1) = 4\theta$

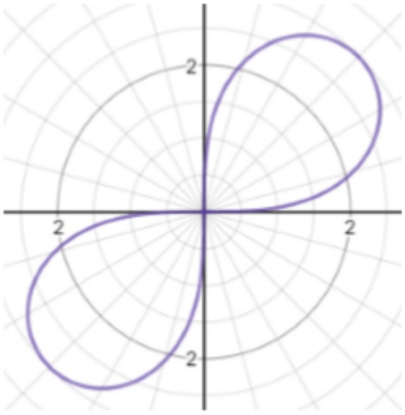
$0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi = 4\theta$ → $\div 4$

add π

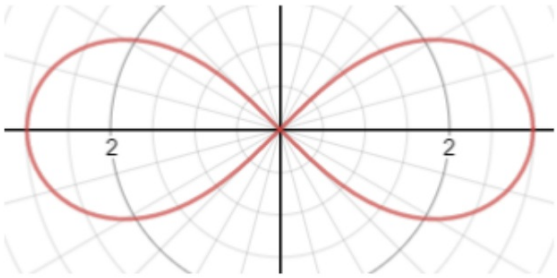
$\theta, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} = \theta$

Lemniscates

$$r^2 = a^2\sin(2\theta)$$



$$r^2 = a^2\cos(2\theta)$$

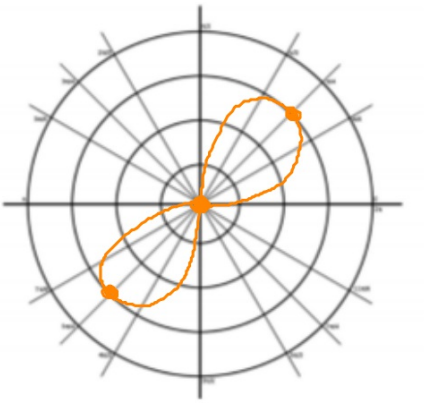


Quick Tips for Graphing Lemniscates

- Cosine graphs are longest along the x axis, this is equivalent to the a value
- Sine graphs are longest along the diagonal, 45 degrees, this is equivalent to the a value

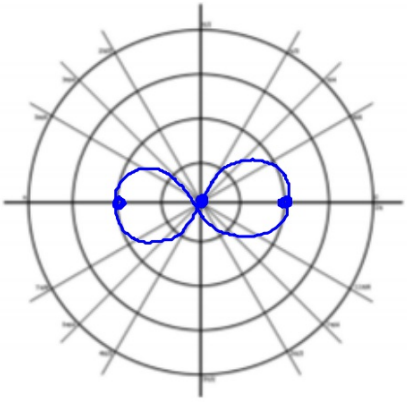
Lemniscates

$r^2 = 9\sin(2\theta)$



θ	0	$\pi/4$	$\pi/2$	$5\pi/4$	π			
r	0	3	0	3	0			

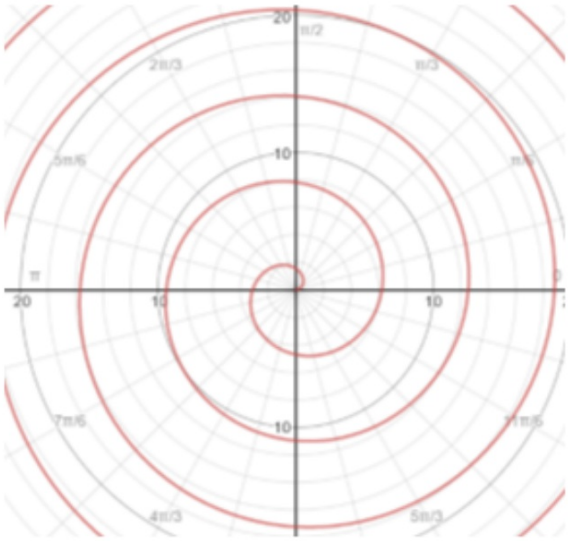
$r^2 = 4\cos(2\theta)$



θ	0	$\pi/2$	π					
r	2	X	2					

Spirals

$$r = \theta$$



(π, π)

$(2\pi, 2\pi)$

$\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$